Attributes

Selective Learning and Influence

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Attribute sampling

This paper revisits a fundamental learning problem:

- agent considers the adoption of an **object of uncertain value**
- object characterized by a mass of **correlated attributes**
- value for the object depends on the **sum** of attribute realizations
- agent might have some **benchmark knowledge**
- **limited sampling** opportunities for additional attributes

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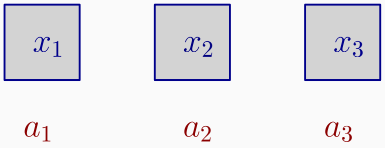

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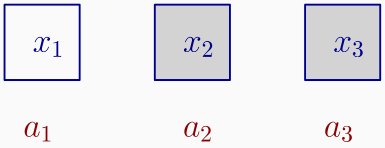

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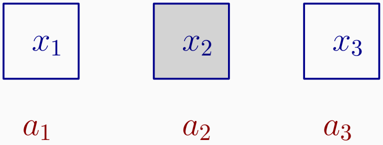

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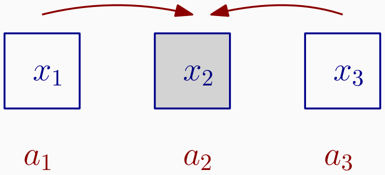

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$$\text{expected value} = \omega_1 \cdot x_1 + \omega_2 \cdot \mathbb{E}[x_2 \mid x_1, x_3] + \omega_3 \cdot x_3$$

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Examples:

- appraising a multi-attribute product before purchase
- evaluating skill bundle of a potential employee
- gauging the spatial impact of a social program

Attribute sampling

Selective exploration of attributes has a long tradition in economics.

Attribute-based demand: Lancaster (1966), Keeney and Raiffa (1976)

Independent attributes: Neeman (1995), Klabjan, Olszewski, and Wolinsky (2014), Sanjurjo (2017)

Our attribute sampling problem significantly differs from the standard:

- search problem
- multi-armed bandit problem

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This paper:

1. Optimal attribute sampling in the absence of agency conflict
2. Distortions in sampling in the presence of agency conflict
   - separate authorities over sampling and adoption
   - different weighting of attributes and/or outside option

Site selection in program evaluation

Selection of pilot sites as attribute sampling

- N target sites (N large)
- sites ordered according to observable characteristics
- program outcomes differ across sites
- learning through small-scale pilot studies ($k << N$)
- program scale-up desirable if average outcome is high
- which sites should be selected for pilot testing?

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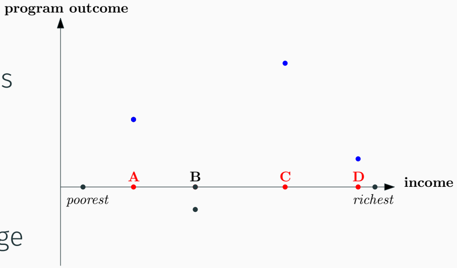

Site selection in program evaluation

Empirical Concern:

Low *generalizability* of pilot findings in impact evaluations

- Allcott (2015), Bold et al. (2018), Vivalt (2020)

- game between a utilitarian researcher and a partisan evaluator
- sufficient statistic for generalizability
- reasonable benchmark for generalizability

- we show the optimal pilot site of low generalizability for both the researcher and the evaluator

Gaussian framework

We model the attribute mapping as a realization of a Gaussian process

- flexible modeling of correlated attributes
- learning over the space of Gaussian sample paths
- covariance function as a similarity metric over pairs of attributes
    - how much can be extrapolated from one attribute to another

The analysis hinges on two key assumptions:

1. Jointly Gaussian attributes
2. Rich attribute space

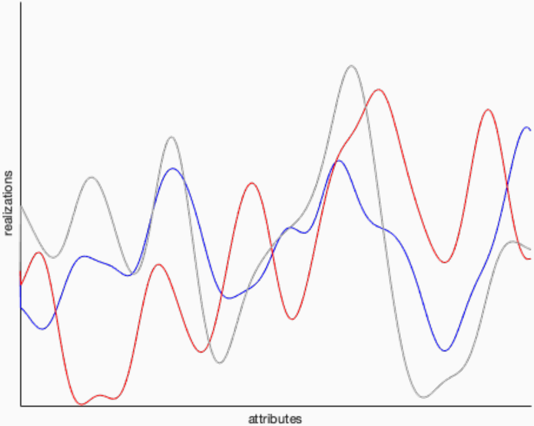

Figure 1: Sample paths of a Gaussian process

Preview of results

1. In the single-player benchmark, the optimal sample
    - maximizes a single informativeness statistic
        - sample balances generalizability to out-of-sample attributes with non-redundancy within sample
        - maximally central in a corresponding attribute graph
    - is independent of expected value of attributes/project
    - is independent of timing format (sequential vs. simultaneous)

Preview of results

2. When agent samples and principal adopts:
    - the value of a sample hinges on two informativeness statistics, one for each player
    - prior agreement between players brings
        - suppression of informativeness for both players
        - controversial sampling
    - distortions in sample size, content, and delay

Model

Players and timing

- Two players: principal (P) and agent (A)
- Players jointly evaluate a multi-attribute project of unknown quality
- Separate authorities:

  $t = 1$: A samples attributes
  $t = 2$: P decides whether to adopt

- Sample observations revealed publicly
  - symmetrically informed players
  - no contracting
- Formats of sampling contrasted
  (i) simultaneous
  (ii) sequential

Attributes

- Attributes $a \in \mathcal{A} := [0, 1]$
- Unknown mapping $f : \mathcal{A} \to \mathbb{R}$ determines attribute realizations
- f drawn from the space of sample paths of a Gaussian process

$$f \sim \mathcal{GP}(\mu, \sigma)$$

where prior mean μ and symmetric positive definite covariance σ:

$$\mu : \mathcal{A} \to \mathbb{R}$$
$$\sigma : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$$

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$$\mu : \mathcal{A} \to \mathbb{R}$$

$$\sigma : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$$

$$\mu(a) = \mathbb{E}[f(a)], \quad \sigma(a, a') = \mathbb{E}\left[(f(a) - \mu(a))\left(f(a') - \mu(a')\right)\right]$$

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- $\sigma(a, a')$ similarity metric among attribute pair (a, a')

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Assumption (Continuity of sample paths)

Almost surely any realization of f is continuous.

Brownian motion is a Gaussian process.

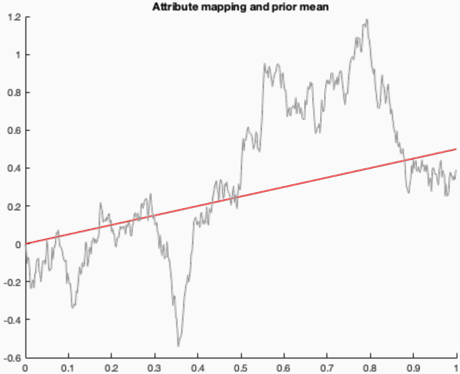

Figure 2: Brownian motion: $\mu = 2a$, $\sigma(a, a') = \min(a, a')$

Sampling

Finite distribution: For any k-sample of attributes $\mathbf{a} = (a_1, \ldots, a_k)$

$$f(\mathbf{a}) := \begin{pmatrix} f(a_1) \\ \vdots \\ f(a_k) \end{pmatrix} \sim \mathcal{N}\left(\underbrace{\begin{pmatrix} \mu(a_1) \\ \vdots \\ \mu(a_k) \end{pmatrix}}_{\mu(\mathbf{a})}, \underbrace{\begin{pmatrix} \sigma(a_1, a_1) & \ldots & \sigma(a_1, a_k) \\ \vdots & \ddots & \vdots \\ \sigma(a_k, a_1) & \ldots & \sigma(a_k, a_k) \end{pmatrix}}_{\Sigma(\mathbf{a})} \right)$$

If $\mathbf{a}$ drawn, $f(\mathbf{a})$ observed perfectly by both players

$\mathcal{A}_k$ is the set of non-redundant samples of size at most k:

$\mathcal{A}_k := \{(a_1, \ldots, a_n) \in \mathcal{A}^n,\ \forall n \leq k, n \in \mathbb{N} \mid \Sigma((a_1, \ldots, a_n)) \text{ is non-singular}\}$

Payoffs

Cost of sampling exogenous (finite) sampling capacity $k \in \mathbb{N}$

$$c(n) = \begin{cases} 0 & \text{if } n \leq k \\ +\infty & \text{otherwise} \end{cases}$$

Rejection payoff heterogenous payoffs from status quo $(r_A, r_P) \in \mathbb{R}^2$

Adoption payoff player i obtains ex-post payoff

$$v_i = \int_{\mathcal{A}} f(a)\omega_i(a)\,\mathrm{d}a$$

where $\omega_i : \mathcal{A} \to \mathbb{R}$ is a Lebesgue-integrable attribute weight function for player i

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Without loss, for both players

$$\int_{\mathcal{A}} \omega_i(a)\,\mathrm{d}a = 1.$$

In the single-player benchmark we normalize $\omega(\cdot) \geq 0$.

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Player i's prior value from the project

$$\nu_0^i := \mathbb{E}[v_i] = \int_{\mathcal{A}} \mu(a)\omega_i(a)\,\mathrm{d}a$$

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$$v_i \sim \mathcal{N}\left(\nu_0^i, \quad \underbrace{\int_{\mathcal{A}}\int_{\mathcal{A}} \sigma(a,a')\omega(a)\omega(a')\,\mathrm{d}a\,\mathrm{d}a'}_{\text{aggregate uncertainty about the project}}\right)$$

Payoffs

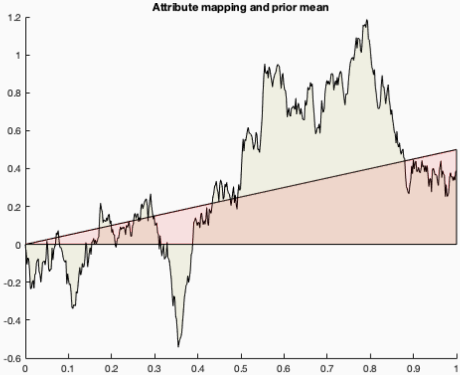

Figure 3: v^i depicted in yellow and ν_0^i in red if $\omega_i(a) = 1$ for all $a \in [0, 1]$

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Sources of conflict:

1. relative importance of attributes (ω_P, ω_A)
2. threshold on adoption (r_A, r_P)

Definition

Players are in *prior disagreement* about the project's initial worth if $\left(v_0^P - r_P\right)$ and $\left(v_0^A - r_A\right)$ have opposite signs.

They are in *prior agreement* otherwise.

Prior disagreement

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They are in *prior agreement* otherwise.

- In the absence of any sampling, prior disagreement implies that players favor different adoption decision.

Underdetermination by data

Assumption

Fix $k \in \mathbb{N}$. For any $\mathsf{a} \in \mathcal{A}_k$, any realization $f(\mathsf{a})$, and at least some player,

$$Var\left[v_i \mid \mathsf{a}, f(\mathsf{a})\right] > 0.$$

Out-of-sample extrapolation

Lemma (Extrapolation)

Fix a sample $\mathsf{a} = (a_1, \ldots, a_k)$ *with respective realizations* $f(\mathsf{a})$ *and attribute* $\hat{a} \in \mathcal{A}$*. The expected realization* $f(\hat{a})$ *is given by*

$$\mathbb{E}[f(\hat{a}) \mid \mathsf{a}, f(\mathsf{a})] = \mu(\hat{a}) + \sum_{j=1}^{k} \tau_j(\hat{a}; \mathsf{a}) \left(f(a_j) - \mu(a_j)\right),$$

where $\tau_j(\hat{a}; \mathsf{a})$*, its sensitivity to observation* $f(a_j)$*, is the* $(1, j)^{th}$ *entry of matrix*

$$\begin{pmatrix} \sigma(a_1, \hat{a}) & \ldots & \sigma(a_k, \hat{a}) \end{pmatrix} \Sigma^{-1}(\mathsf{a}).$$

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$$\begin{pmatrix} \sigma(a_1, \hat{a}) & \ldots & \sigma(a_k, \hat{a}) \end{pmatrix} \Sigma^{-1}(\mathsf{a}).$$

- predicted realization for any attribute is a linear combination of sample realizations
- $\tau(\hat{a}; a_j) \equiv$ extent to which $f(a_j)$ contributes to the guess for $f(\hat{a})$
- exact shape of extrapolation depends on covariance σ

Examples: $\mu(a) = 0$, $\mathbf{a} = (1/5, 2/5, 3/5, 4/5)$

Let's see a few examples of extrapolation from a sample.

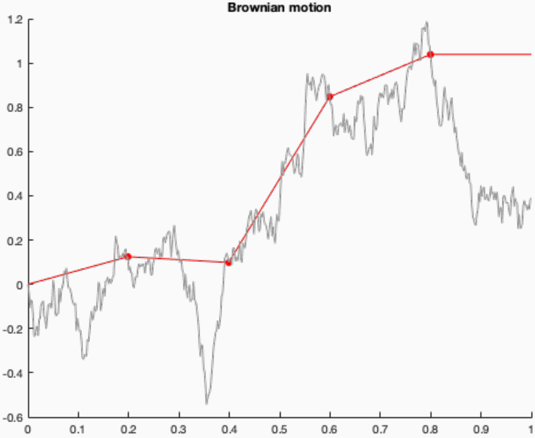

Figure 4: Brownian: $\sigma(a, a') = \min(a, a')$

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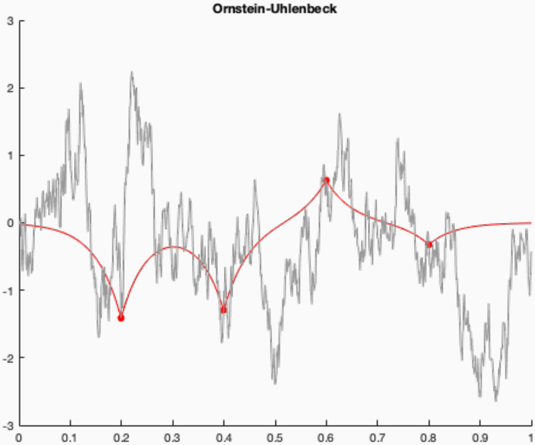

Figure 4: Ornstein-Uhlenbeck: $\sigma(a, a') = e^{-|a-a'|/\ell}$, $\ell = 1/20$

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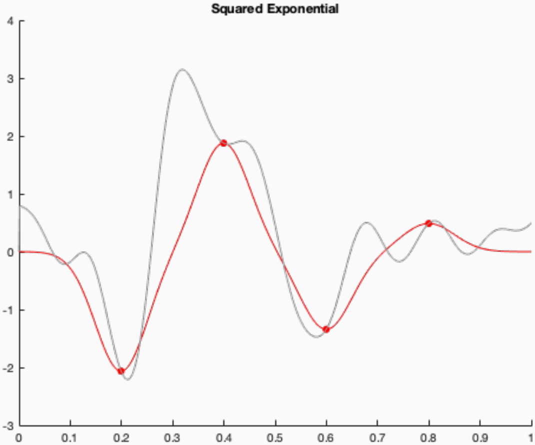

Figure 4: Squared exponential: $\sigma(a, a') = e^{-(a-a')^2/\ell^2}$, $\ell = 1/20$

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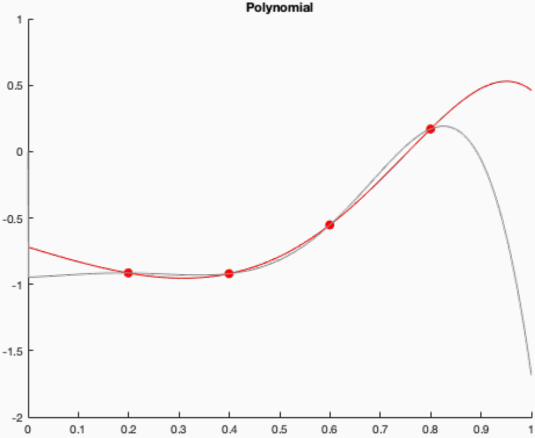


Figure 4: Polynomial: $\sigma(a, a') = (1 + aa')^{10}$

Posterior value

Lemma

Fix sample $\mathsf{a} = (a_1, \ldots, a_k)$ *with respective realizations* $f(\mathsf{a})$. *Player i's posterior value is a linear combination of sample realizations, i.e.*

$$\nu^i(\mathsf{a}, f(\mathsf{a})) = \nu_0^i + \sum_{j=1}^{k} \tau_j^i(\mathsf{a}) \left(f(a_j) - \mu(a_j)\right)$$

where realization $f(a_j)$ *is weighted by*

$$\tau_j^i(\mathsf{a}) := \int_{\mathcal{A}} \tau_j(a; \mathsf{a}) \omega_i(a) \, \mathrm{d}a$$

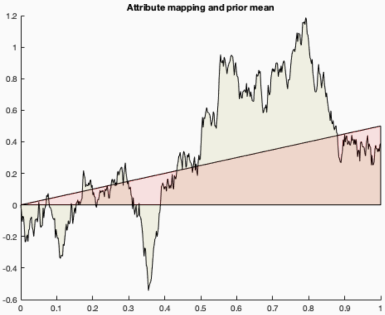
and $\tau_j(a; \mathsf{a})$ *is as above.*

- sensitivity of posterior to $f(a_j)$ aggregates sensitivity of the entire extrapolated mapping to it

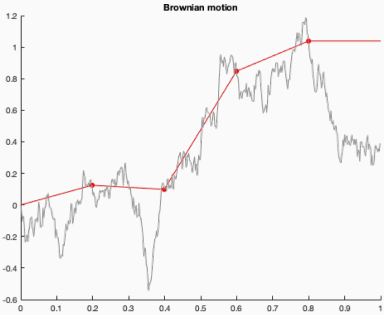
Related work

1. **Attribute discovery and selective information gathering**: Neeman (1995), Branco, Sun and Villas-Boas (2012), Klabjan, Olszewski, Wolinsky (2014), Olszewski and Wolinsky (2016), Sanjurjo (2017), Che and Mierendorff (2017), Liang, Mu, Syrgkanis (2020)
2. **Learning and experimentation over Gaussian paths**: Jovanovic and Rob (1990), Aghion, Bolton, Harris, and Jullien (1991), Callander (2011), Callander and Hummel (2014), Garfagnini and Strulovici (2016), Ilut and Valchev (2017)
3. **Persuasion through constrained experimental design**: Glazer and Rubinstein (2004), Brocas and Carrillo (2007), Rayo and Segal (2010), Hirsch (2016), Banerjee, Chassang, Montero, Snowberg (2017), Di Tillio, Ottaviani, Sorensen (2017)
4. **GPs in geostatistics and machine learning**: Matheron (1963, 1967), Chilés and Delfiner (2012), Rasmussen and Williams (2006)

- Payoff structure: finding a maximum vs. estimating the area
- Gaussian process approach allows us to bypass invoking the Brownian bridge



(a)



(b)

I. Single-player sampling

Ranking of samples

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  - index i dropped

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$$\nu_0 = \int_0^1 \mu(a)\omega(a)da$$

- Posterior value is Gaussian

$$\nu(\mathsf{a}, f(\mathsf{a})) \sim \mathcal{N}(\nu_0, \psi^2(\mathsf{a}))$$

- $f(\mathsf{a})$ does not enter posterior variance $\psi^2(\mathsf{a})$
- Samples ranked according to $\psi^2(\mathsf{a})$

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- Agent's expected payoff from sample a:

$$V(\mathsf{a}) = r + (\nu_0 - r)\Phi\left(\frac{\nu_0 - r}{\psi(\mathsf{a})}\right) + \psi(\mathsf{a})\phi\left(\frac{\nu_0 - r}{\psi(\mathsf{a})}\right)$$

  V strictly increasing and convex in ψ

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Optimal sampling

Theorem (Single-player sampling)

Fix $k \in \mathbb{N}$. Any single-player sample a^*

(i) *consists of k distinct attributes;*

(ii) *maximizes posterior variance $\psi^2(\cdot)$, given by*

$$\mathsf{a}^* \in \arg\max_{\mathsf{a} \in \mathcal{A}_k} \sum_{j=1}^{k} \sum_{m=1}^{k} \tau_j(\mathsf{a})\tau_m(\mathsf{a})\sigma(a_j, a_m) := \psi^2(\mathsf{a});$$

(iii) *is independent of μ, ν_0, and r.*

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- only covariance and attribute weights enter into ψ
- two attributes reinforce each other in the sample if

$$\tau_j(\mathsf{a})\tau_m(\mathsf{a})\sigma(a_j, a_m) > 0$$

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(i) *consists of k distinct attributes;*

(ii) *maximizes posterior variance $\psi^2(\cdot)$, given by*

$$\mathsf{a}^* \in \arg\max_{\mathsf{a} \in \mathcal{A}_k} \sum_{j=1}^{k} \sum_{m=1}^{k} \tau_j(\mathsf{a}) \tau_m(\mathsf{a}) \sigma(a_j, a_m) := \psi^2(\mathsf{a});$$

(iii) *is independent of μ, ν_0, and r.*

Proposition (Equivalence of sampling formats)

A sample is optimal under sequential sampling of attributes if and only if it is optimal under simultaneous sampling.

Site selection: Researcher’s benchmark

Implications for site selection

Let us reinterpret attributes as sites and f as outcome of the program.

A utilitarian researcher weighs all sites equally:

$$\omega_A(a) = 1 \quad \forall a \in [0, 1]$$

Which sites would the researcher select if in charge of program adoption as well?

1. Site selection is unbiased from expected outcomes
2. $\psi(\mathbf{a})$ as a measure of external validity of sample sites $\mathbf{a}$
3. Timing of pilots is immaterial: early vs. late pilots

Distance-based covariance

Suppose site outcomes are correlated according to

$$\sigma_{OU}(a, a') = e^{-|a-a'|/\ell}$$

where ℓ is a length-scale parameter.

We normalize $\mu(a) = 0$ for all $a \in [0, 1]$.

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- distance-based covariance
- ℓ measures correlation across a fixed distance
  - $\ell \to 0$: independent outcomes
  - $\ell \to +\infty$: perfectly correlated outcomes
- all site outcomes are ex ante identical

$$f(a) \sim \mathcal{N}(0, 1) \quad \text{for all } a \in [0, 1]$$

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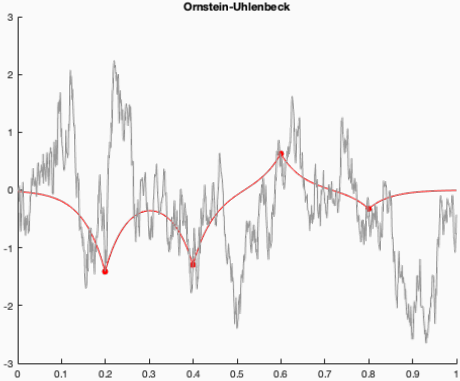

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We normalize $\mu(a) = 0$ for all $a \in [0, 1]$.

This covariance is highly tractable $\Rightarrow$ closed-form $\tau_j(\mathsf{a})$ and $\psi^2(\mathsf{a})$

The researcher's optimal sample:

- unique and symmetric around the median site 1/2
- each sample site is weighted equally
- more dispersed as correlation strengthens (i.e., $\ell \uparrow$)
- leftmost site pinned down by

$$1 - e^{-a_1^*/\ell} = \tanh\left(\frac{1 - 2a_1^*}{2\ell(k-1)}\right)$$

Varying ℓ

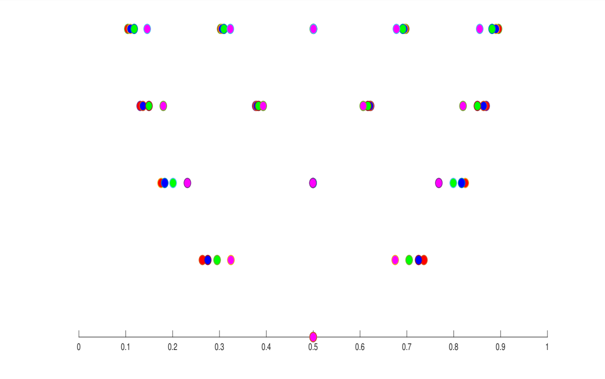

Figure 5: The researcher's sample illustrated for $k \in \{1, \ldots, 5\}$ (bottom up) and $\ell = 1$, $\ell = 1/2$, $\ell = 1/5$, $\ell = 1/20$.

Varying ℓ

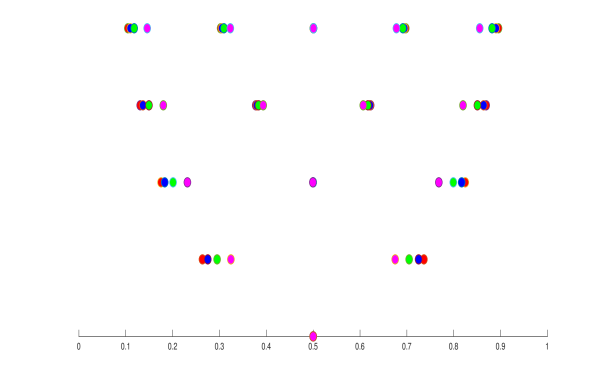

Figure 5: The researcher's sample illustrated for $k \in \{1, \ldots, 5\}$ (bottom up) and $\ell = 1$, $\ell = 1/2$, $\ell = 1/5$, $\ell = 1/20$.

As sites become independent ($\ell \to 0$), sample converges to

$$\left(\frac{1}{k+1}, \ldots, \frac{k}{k+1} \right)$$

Varying ℓ

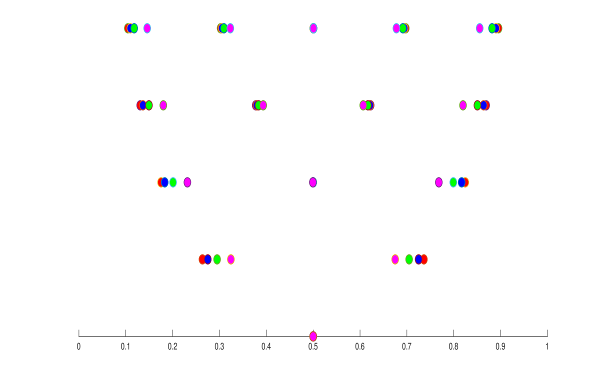

Figure 5: The researcher's sample illustrated for $k \in \{1, \ldots, 5\}$ (bottom up) and $\ell = 1$, $\ell = 1/2$, $\ell = 1/5$, $\ell = 1/20$.

As sites become perfectly correlated ($\ell \to +\infty$), sample converges to

$$\left(\frac{1}{2k}, \ldots, \frac{2k-1}{2k}\right)$$

Sample centrality

Centrality of a sample

In the previous example, the optimal sample is central in $[0, 1]$.

Is there a formal sense in which the optimal sample is most central in the attribute space for any (ω, σ)?

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Centrality of a sample

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Is there a formal sense in which the optimal sample is most central in the attribute space for any (ω, σ)?

- Yes, the optimal sample maximizes sample centrality
- Generalization of betweenness centrality to sets of nodes
- expected walk sum from a random attribute a to another a' such that each walk traverses sample attributes only
- random pair (a, a') drawn according to density $\omega(a)\omega(a')$

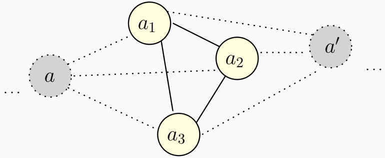

Figure 6: Sample $\mathbf{a} = (a_1, a_2, a_3)$

Sample centrality

II. Principal - agent sampling

Agent's payoff

- Prior disagreement: $\left(\nu_0^A - r_A,\ \nu_0^P - r_P\right)$
- Upon sampling $(\mathsf{a}, f(\mathsf{a}))$ principal adopts iff

$$\nu^P\left(\mathsf{a}, f(\mathsf{a})\right) \geq r_P$$

- $\rho(\mathsf{a}) \equiv$ correlation of posteriors $\nu^P(\mathsf{a})$ and $\nu^A(\mathsf{a})$
  - If ω_i is the same for both players, $\rho(\mathsf{a}) = 1$ for any sample a

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Agent's expected payoff from $\mathbf{a}$

$$r_A + \underbrace{\Pr\left(\nu^P(\mathbf{a}) \geq r_P\right)}_{\text{probability of adoption}} \cdot \left(\underbrace{\mathbb{E}\left[\nu^A(\mathbf{a}) \mid \nu^P(\mathbf{a}) \geq r_P\right]}_{\text{inference from adoption}} - r_A\right)$$

Agent's payoff

Theorem (Sufficient statistics for a sample)

For any sample a, *the agent's expected payoff depends on* a *only through the pair of sufficient statistics*

$$(\alpha_1(\mathsf{a}), \alpha_2(\mathsf{a})) := (\psi_P(\mathsf{a}), \quad \rho(\mathsf{a})\psi_A(\mathsf{a}))$$

where ψ_i *denotes posterior variance for player i. All else fixed, his payoff is strictly increasing in* α_2.

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Pair (α_1, α_2) summarizes how informative a is for each player.

- $\alpha_1(\mathsf{a})$: how informative is the sample in single-principal benchmark
- $\alpha_2(\mathsf{a})$: how informative is adoption for the agent
  - informativeness ψ_A adjusted by correlation ρ
  - its sign same as the sign of ρ
  - share of the posterior variance for the agent that gets reflected in the adoption decision

Agent's payoff

Agent's expected payoff from sample $\mathbf{a}$

$$r_A + \underbrace{(\nu_0^A - r_A)\Phi\left(\frac{\nu_0^P - r_P}{\alpha_1(\mathbf{a})}\right)}_{\text{adoption probability}} + \underbrace{\alpha_2(\mathbf{a})\phi\left(\frac{\nu_0^P - r_P}{\alpha_1(\mathbf{a})}\right)}_{\text{adoption accuracy}}$$

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Adoption probability $\nearrow$ in α_1 iff prior disagreement

- e.g., suppose $\nu_0^P > r_P$
- more informative for principal $\Rightarrow$ less adoption $\Rightarrow$ preferred if $\nu_0^A < r_A$

Agent's payoff

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Adoption accuracy $\nearrow$ in α_1 fixing $\alpha_2 > 0$

- better informed principal implies better informed adoption decision
- this goes in agent's favor iff sample aligns their interests: $\rho > 0$

Tradeoff between the two considerations is pinned down by (ν_0^A, ν_0^P).

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Remark

Sequential sampling strictly preferred by the agent.

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Remark

Under prior agreement, agent seeks to increase both statistics as much as possible.

Two distortions

1. **Joint suppression of informativeness**

   Is it ever optimal to select a sample that is dominated in both informativeness statistics?

   A sample $\mathsf{a} \in \mathcal{A}_k$ is *dominated* if there exists another $\mathsf{a}' \in \mathcal{A}_k$ such that $\alpha_i(\mathsf{a}') \geq \alpha_i(\mathsf{a})$ with strict inequality for some $i = 1, 2$.

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▸ Propositions

Site selection: Strategic suppression

Strategic site selection

- Covariance $\sigma_{OU}(a, a')$ for all $a, a' \in [0, 1]$
- Partisan evaluator P vs. utilitarian researcher A
- Site weights

$$\omega_P(a) = \begin{cases} +\infty & \text{for } a = a_P \\ 0 & \text{for } a \neq a_P \end{cases} \qquad \Bigg| \qquad \omega_A(a) = 1 \quad \forall a \in [0, 1]$$

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- Prior values of researcher and evaluator respectively:

$$\text{Average outcome}: \quad \nu_0^A = \int_0^1 \mu(a)\, \mathrm{d}a =: \bar{\mu}$$

$$\text{Partisan outcome}: \quad \nu_0^P = \mu(a_P)$$

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- Where to place a single pilot ($k = 1$)?

For any sample site $a \in [0, 1]$, the induced $\rho(a) = 1$. Hence,

$$(\alpha_1(a),\ \alpha_2(a)) = (\psi_P(a),\ \psi_A(a))$$

Single-player sites

$$a_P^* = a_P \geq 1/2, \quad a_A^* = 1/2$$

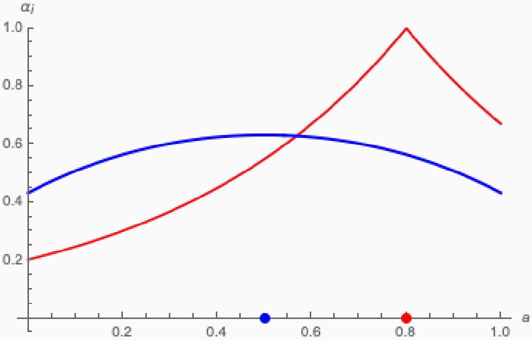

Figure 7: ψ_P in red and ψ_A in blue

1. Compromise in site selection

Sites in $[1/2, a_P]$ are compromise sites

- strict trade-off between posterior variances

Proposition

(i) *If players are in prior disagreement, the optimal site is a compromise.*

(iii) *Suppose prior disagreement and fix* $\bar{\mu}$. *The optimal site is increasing in* $|\mu(a_P)|$. *If* $\mu(a_P) = 0$, *the optimal site is the median site. For sufficiently large* $|\mu(a_P)|$, *the optimal site is exactly* a_P.

2. Optimal selection of peripheral sites

- Prior agreement is necessary for the optimal site $a^* \notin [1/2, a_P]$

Proposition

Suppose players are in prior agreement. For $a_P > 1/2$ and $\mu(a_P)$ sufficiently close to zero, the optimal site is unique and $a^* < 1/2$.

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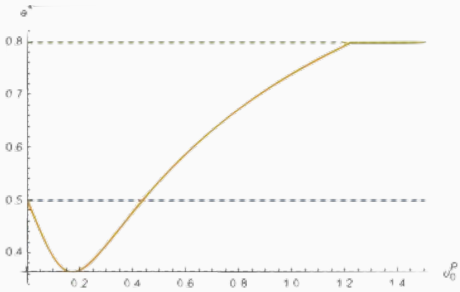

Figure 8: $\mu(a_P)$ in x-axis and optimal site in y-axis. $\ell = 1/2, \bar{\mu} = 1/2, a_P = 4/5$.

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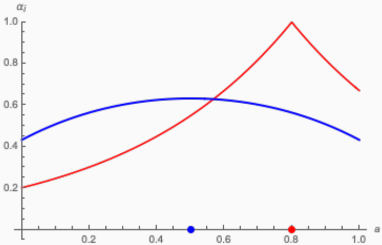

Figure 8: $a_P = 4/5$

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Proposition

Suppose players are in prior agreement. For $a_P > 1/2$ and $\mu(a_P)$ sufficiently close to zero, the optimal site is unique and $a^* < 1/2$.

- at $\mu(a_P) = 0$, the median site is optimal
- for $|\mu(a_P)|$ sufficiently small relative to $|\bar{\mu}|$, influencing the probability of adoption is of first order
- suppressing ψ_P preserves evaluator's prior bias
- this come at a cost for researcher: suppress ψ_A too

This continues to hold even if $a_P = 1/2$: median site is most informative for both researcher and evaluator

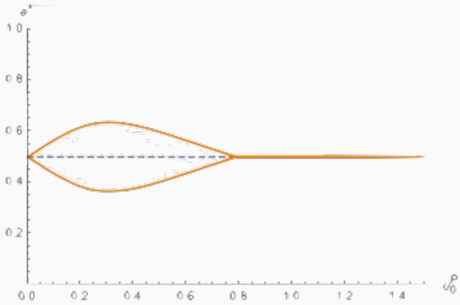

Figure 8: $\mu(a_P)$ in x-axis and optimal site in y-axis. Parameter values: $\ell = 1/2, \bar{\mu} = 1/2, a_P = 1/2$.

3. Distortions largest for moderate correlation

- Distortions even when $a_A = a_P = 1/2$
- Distortions vanish as $\ell \to +\infty$ and $\ell \to 0$
- $|a^* - 1/2|$ single-peaked in ℓ

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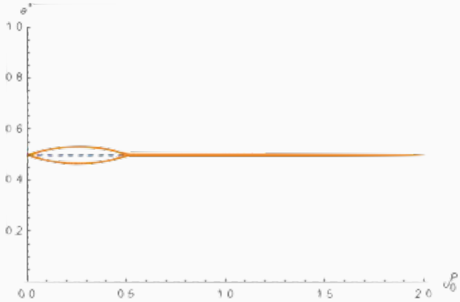

Figure 9: $\mu(a_P)$ in x-axis and optimal site in y-axis. Parameter values: $\ell = 10, \bar{\mu} = 1/2, a_P = 1/2$.

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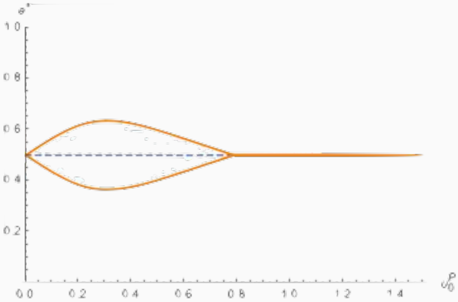

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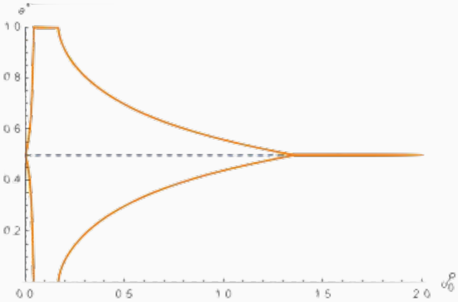

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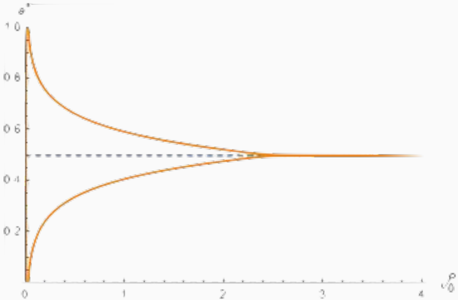

Figure 9: $\mu(a_P)$ in x-axis and optimal site in y-axis. Parameter values: $\ell = 1/10, \bar{\mu} = 1/2, a_P = 1/2$.

Concluding remarks

Discussion

Flexible framework for modeling selective sampling of attributes

1. selective learning:
   simple informativeness index for identifying the single-player sample
2. influence:
   taxonomy of distortions due to the sample controlling both learning and alignment of players

Discussion

Tractable and novel learning framework to further address:

- Partial / targeted adoption
  - the attribute problem is inherently one of full scale or no adoption
  - adoption of a strict subset of attributes upon inspection
  - bridge between problems of search and attribute sampling
- Aggregation of local knowledge
  - constrained access to attribute realizations (site outcomes)
  - sites / attributes need to be incentivized to collect and/or impart local information

Thank you!

Centrality of a sample

Heuristic construction: centrality of a sample in the attribute graph

- without loss, $\omega(a) \geq 0$ for all $a \in \mathcal{A}$
- reminder: weights add to up
- $\sigma(a, a) = 1$ for all $a \in \mathcal{A}$

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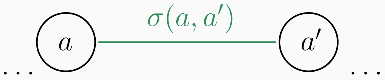
▶ Infinite weighted graph $\mathcal{G}(\mathcal{A}, E)$ with

- attributes as nodes
- edge weight $e_{aa'} = \sigma(a, a')$



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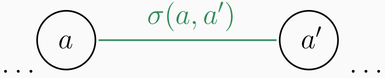
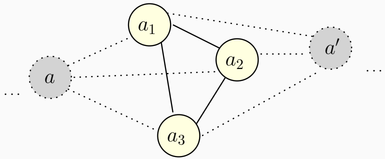


▶ $\mathcal{G}_{\mathbf{a}}$: subgraph consisting of nodes $\mathbf{a}$ and edges within

Sample centrality



Sample centrality is a function

$$\gamma : \bigcup_{k \in \mathbb{N}} \mathcal{A}_k \to \mathbb{R}$$

equal to

- the sum of walks of any length ...
- from a random node $a \in \mathcal{A}$ ...
- to another random node a' ...
- drawn according to density $\omega(a)\omega(a')$ ...
- such that all intermediate nodes in each walk are in $\mathcal{G}_{\mathsf{a}}$.

Akin to betweenness centrality for non-singleton sets of nodes.

Sample centrality

Theorem (Sample centrality of a single-player sample)

(i) *For any sample* a *for which* $\mathcal{G}$ *is* a*-walk-summable, its sample centrality is equal to the posterior variance that the sample induces, i.e.* $\gamma(\mathsf{a}) = \psi^2(\mathsf{a})$.

(ii) *Fix capacity* k*, and suppose* $\mathcal{G}$ *is* k*-walk-summable. Any single-player sample attains the highest sample centrality.*

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If walk-summability fails, we modify it through path-summability

- (finite-length) paths instead of walks within $\mathcal{G}_{\mathsf{a}}$
- well-defined for any positive definite covariance σ

▸ Sample centrality

Two distortions

Proposition (Joint suppression of informativeness)

(1) *An optimal sample is dominated only if players are in prior agreement.*

(2) *If all the following hold:*

   (i) *the players are in prior agreement,*

   (ii) *there exists at least one sample* $\mathsf{a} \in \mathcal{A}_k$ *such that* $\rho(\mathsf{a}) > 0$,

   (iii) *at any* α_2*-maximal feasible sample, there exists a sample arbitrarily close to it that is dominated by it*

then there exist $\bar{x}^P$ *and* $\underline{x}^A < \bar{x}^A$ *such that for* $|\nu_0^P - r_P| \leq \bar{x}^P$ *and* $\underline{x}^A \leq |\nu_0^A - r_A| \leq \bar{x}^P$ *any optimal sample* a^* *is dominated and has* $\alpha_1(\mathsf{a}^*) > 0$.

Proposition (Influence via controversial sampling)

(i) *A controversial sample is optimal only if players are in prior disagreement.*

(ii) *When in prior disagreement, agent forgoes informative sampling if and only if all feasible samples are controversial and* $\rho(\mathsf{a})$ *is sufficiently negative for all* $\mathsf{a} \in \mathcal{A}_k$.

(iii) *If the optimal sample* a^* *is controversial, then for any feasible non-controversial sample* a, $\psi_P(\mathsf{a}^*) > \psi_P(\mathsf{a})$.

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