Economics 690 – Spring 2023

Continuous-time methods in economic theory

Instructor	Arjada Bardhi
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Class meetings	WF 5:15-6:30 PM, Perkins LINK 088 (Classroom 4)
Office hours	Friday, 10 AM - noon, SocSci 223 / Zoom (sign-up through Sakai)

Course description

In economics and finance, continuous-time techniques have found wide application beyond the celebrated Black-Scholes option pricing formula. They are used in dynamic models of learning and experimentation, contracting, reputation building, capital structure, financial frictions and macroeconomic crises, trade and information disclosure, among others. The main goal of this course is to prepare students to effectively follow and interact with the economic theory literature that makes use of continuous-time techniques. The course is organized in two parts. The first part will present useful concepts and tools in stochastic calculus and stochastic control, with a heavy emphasis on the Brownian motion. The second part will cover select applications from economic theory.

Textbook / references:

- *Brownian Models of Performance and Control*, J. Michael Harrison, Cambridge University Press, 2013. (*)

- *The Economics of Inaction: Stochastic Control Models with Fixed Costs*, Nancy L. Stokey, Princeton University Press, 2009. (*)

- *The Art of Smooth Pasting*, Avinash Dixit, Routledge: Taylor & Francis Group, 2001.

- *Continuous-Time Models in Corporate Finance, Banking, and Insurance: A User's Guide*, Santiago Moreno-Bromberg and Jean-Charles Rochet, Princeton University Press, 2018. (*)

- *Elementary Stochastic Calculus with Finance in View*, Thomas Mikosch, World Scientific Publishing, 1998.

- *Stochastic Calculus for Finance II: Continuous-time Models*, Steven E. Shreve, Springer, 2004.

- *Stochastic Differential Equations: An Introduction with Applications*, Bernt Øksendal, Springer, 2003.

Note: (*) indicates main textbooks. Lecture slides and other relevant materials will be provided through Sakai.

Prerequisites

The course material does not assume familiarity with measure theory, neither will it aim to cover any formally. It also does not attempt to replace a rigorous course in stochastic calculus. The course presupposes a first course in probability theory, analysis, and some degree of familiarity with differential equations. For a quick refresher, interested students are encouraged to read sections 1.1 and 1.4 of Mikosch (1998), chapter 1 in Shreve (2004), chapter 2 and appendices A and B in Øksendal (2003).

Grading policy

Evaluation is based on four biweekly written problem sets (40 %), a short group presentation on the group's topic of choice (20 %), occasional quizzes (10 %), and a final exam (30 %). Problem sets will be posted a week prior to their due date. Students are allowed to work in groups, but they are required to submit individual answers. Problem sets will be submitted through Sakai. Student presentations will be scheduled during the last two weeks of the semester—more details will be given later in the semester. The take-home exam will take place on May 2 at 2-5 PM.

Tentative schedule of topics

Week 1:

Jan 13: Stochastic processes and equivalence notions, filtrations, martingales, stopping times.

Week 2:

Jan 18: Basic properties of the Brownian motion

Jan 20: Quadratic variation; strong Markov property; occupancy measure and local time; Brownian martingales; the Innovation Theorem

Week 3:

Jan 25: Reflection principle and other properties of the Brownian; regular diffusions; Gaussian processes; geometric Brownian; Brownian bridge

Jan 27: Itô integral for simple processes

Assigment 1 due on February 1 at 5:15 PM.

Week 4:

Feb 1: Approximation of the Itô integral for general processes; Itô Isometry

Feb 3: Five variations of Itô's lemma; Itô processes

Week 5:

Feb 8: Tanaka's formula; solutions of SDEs; Optional Stopping Theorem; Martingale Convergence Theorem

Feb 10: Introduction to Lévy processes; simple and compound Poisson processes and their properties; stochastic integral and Itô's lemma with jumps

Week 6:

Feb 15: Optional Sampling Theorem; application to Bayesian learning

Feb 17: Martingale Convergence Theorem; Black-Scholes option pricing

Assigment 2 due on February 22 at 5:15 PM.

Week 7:

Feb 22: Useful solutions for Brownian motion (Stokey, chapter 5)

Feb 24: Expected (discounted) local time; elements of a stochastic control problem; Markov control; dynamic programming and the HJB equation

Week 8:

March 1: Verification theorems; Merton's portfolio allocation problem

March 3: Verification in optimal stopping problems; value matching and smooth pasting

Week 9:

March 8: McDonald and Siegel (1986); Investment with costly waiting

March 10: Stokey's expected local time approach to stopping problems

Spring break March 10-20.

Week 10:

March 22: Menu cost pricing models; HJB approach to impulse control; random costless adjustment

March 24: Inventory models with holding and adjustment costs; long-run averages

Assigment 3 due on March 22 at 5:15 PM.

Week 11:

March 29: Regulated Brownian motion; instantaneous control with discounting; inventory model with no fixed costs

March 31: Dumas (1991); super contact condition; investment with linear and convex adjustment costs

Week 12:

Apr 5: Brownian models of dynamic inference (Harrison, chapter 8)

Apr 7: Student Presentations I

Week 13:

Apr 12: Student Presentations II

Apr 14: Student Presentations III

Assigment 4 due on April 12 at 5:15 PM.

Week 14:

Apr 19: Student Presentations IV

Take-home exam due on May 2 at 5 PM.

Additional reading

Ambrus, Attila and Shih En Lu. "A Continuous-Time Model of Multilateral Bargaining." *American Economic Journal: Microeconomics* 7(1): 208-49, 2015.

Bentolila, Samuel and Giuseppe Bertola. "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *The Review of Economic Studies* 57(3): 381-402, 1990.

Board, Simon and Moritz Meyer-ter-Vehn. "Reputation for Quality." *Econometrica* 81(6): 2381-2462, 2013.

Bolton, Patrick and Christopher Harris. "Strategic Experimentation." *Econometrica* 67(2): 349-374, 1999.

Bonnati, Alessandro and Johannes Hörner. "Collaborating." *American Economic Review* 101: 632 - 663, 2011.

Brunnermeier, Markus K. and Yuliy Sannikov. "A Macroeconomic Model with a Financial Sector." *American Economic Review* 104(2): 379-421, 2014.

Chan, Jimmy, Alessandro Lizzeri, Wing Suen, and Leeat Yariv. "Deliberating Collective Decisions." *Review of Economic Studies* 85(2): 929-963, 2018.

Chaves, Isaias. "Privacy in Bargaining: The Case of Endogenous Entry." Unpublished manuscript, 2019.

Cisternas, Gonzalez. "Two-sided learning and the ratchet principle." *The Review of Economic Studies*, 85(1): 307-351, 2018.

Daley, Brendan and Brett Green. "Bargaining and News." Unpublished manuscript, August 2018.

Daley, Brendan and Brett Green. "Waiting for News in the Market for Lemons." *Econometrica* 80(4): 1433-1504, 2012.

Demarzo, Peter M. and Yuliy Sannikov. "Learning, Termination, and Payout Policy in Dynamic Incentive Contracts." *The Review of Economic Studies* 84 (1), 2017.

Dixit, Avinash. "Entry and Exit Decisions under Uncertainty." *Journal of Political Economy* 97(3): 620-638, 1989.

Dumas, Robert. "Super Contact and Related Optimality Conditions." *Journal of Economic Dynamics and Control* 15: 675-685, 1991.

Fudenberg, Drew, Gleb Romanyuk, and Philipp Strack. "Active Learning with Misspecified Beliefs." *Theoretical Economics* 12(3): 1155-1189, 2017.

Keller, Godfrey, Sven Rady and Martin Cripps. "Strategic Experimentation with Exponential Bandits." *Econometrica* 74(1): 39-68, 2005.

Keller, Godfrey, and Sven Rady. "Strategic Experimentation with Poisson Bandits." *Theoretical Economics* 5: 275 - 311, 2010.

Kim, Kyungmin. "Information About Sellers' Past Behavior in the Market for Lemons." *Journal of Economic Theory*, 169: 365-399, 2017.

Leland, Hayne E. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure." *Journal of Finance* 49(4): 1213-1252, 1994.

McDonald, Robert and Daniel Siegel. "The Value of Waiting to Invest." *The Quarterly Journal of Economics* 101(4): 707-727, 1986.

Moscarini, Giuseppe and Lones Smith. "The Optimal Level of Experimentation." *Econometrica* 69(6): 1629 - 1644, 2001.

Ortner, Juan. "A Continuous-Time Model of Bilateral Bargaining." Working paper, December 2018.

Perry, Motty and Phil Reny. "A Non - Cooperative Bargaining Model with Strategically Timed Offers." *Journal of Economic Theory* 59(1): 50-77, 1993.

Pindyck, Robert S. "Irreversible Investment, Capacity Choice, and the Value of the Firm." *American Economic Review* 78(5): 969-985, 1988.

Pindyck, Robert S. "Irreversibility, Uncertainty, and Investment." *Journal of Economic Literature* 29: 1110-1148, 1991.

Prat, Jullien and Boyan Jovanovic. "Dynamic Contracts When the Agent's Quality is Unknown." *Theoretical Economics* 9: 865-914, 2014.

Rochet, J-C and S. Villeneuve. "Corporate Portfolio Management." *Annals of Finance* 1(3): 225-243, 2005.

Rogerson, Richard, Robert Shimer, and Randall Wright. "Search-Theoretic Models of the Labor Market: A Survey" *Journal of Economic Literature* 43: 959988, 2005.

Sannikov, Yuliy. "A Continuous-Time Version of the Principal-Agent Problem." *The Review of Economic Studies*, 75(3): 957 - 984, 2008.

Strulovici, Bruno and Martin Szydlowski. "On the Smoothness of Value Functions and the Existence of Optimal Strategies in Diffusion Models." *Journal of Economic Theory* 159: 1016-1055, 2015.

Strulovici, Bruno. "Learning While Voting: Determinants of Collective Experimentation." *Econometrica* 78(3): 933-971, 2010.